

increases. Furthermore, the possibility of increasing the number of variable parameters arises, which makes the trial functions more flexible without noticeably increasing the volume of computations.

The method continues to work quite effectively in those cases when the number of unknown functions cannot be reduced, although perhaps slightly less accurately.

NOTATION

V_i , H_i , components of velocity of center of mass and magnetic field intensity; ρ , density; p , pressure; μ , magnetic susceptibility; T , temperature; κ , thermal diffusivity; σ , electrical conductivity; α , coefficient of thermal expansion; λ , vector with components (0, 0, 1); ∇ , Hamiltonian operator; ΔT , temperature drop between boundaries.

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EFFECTS OF THERMOELASTIC STRESS DURING HONING ON COMPONENT DIMENSIONS

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Equations are given for the temperature distribution and thermoelastic deformations arising in the workpiece and tool during honing, as well as a method for determining the corrections for the temperature errors in adjusting automatic-control devices.

The uses of honing are extending continually; when the process is introduced, it is usually necessary to provide automatic monitoring of workpiece dimensions.

In turn, the setting-up procedure for the monitoring requires information about the errors arising from thermal expansion of the workpiece and the tool, since the latter can be very important, especially for thin-walled components.

The thermal deformation can be determined from the temperature distribution produced in the workpiece 1 and honing rod 2 (Fig. 1), with subsequent calculation of the deformation on the basis of the theory of elasticity.

The hone rotates at a relatively high speed, and the area of contact with the workpiece is fairly large, so one can assume that the temperature over the entire inner surface of the component is the same to a first approximation and equal to the contact temperature θ_c . We may also suppose that the outer surface A of the workpiece 1 (Fig. 1) has boundary conditions of the third kind, since this surface is usually cooled by a continuous flow of liquid.

The temperature of the coolant (kerosene) and the initial temperature of the workpiece may be taken as nominally zero.

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TABLE 1. Values of C_n

n	$Bi=5,05,$ $k=0,5$	$Bi=4,2$ $k=0,6$	n	$Bi=5,05,$ $k=0,5$	$Bi=4,2,$ $k=0,6$
1	-0,56	-0,818	5	-0,172	-0,207
2	+0,317	+0,358	6	+0,061	+0,05
3	-0,273	-0,326	7	-0,126	-0,155
4	+0,129	+0,126	8	+0,025	+0,01

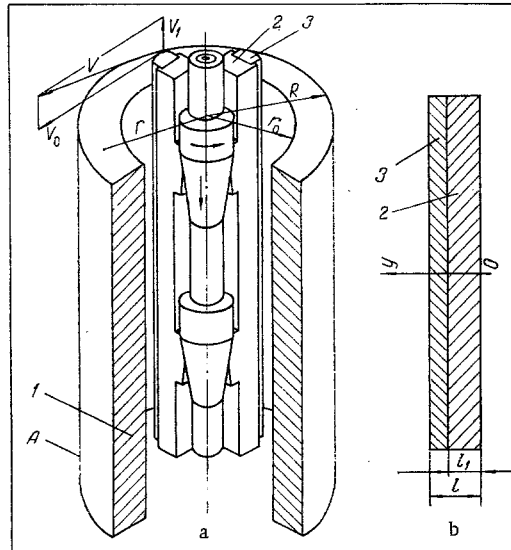


Fig. 1. Honing scheme: 1) workpiece; 2) holder with hone 3.

Under the above conditions, the temperature distribution in the workpiece can be represented as dependent only on the radius r and as satisfying the thermal-conduction equation:

$$\frac{\partial \theta}{\partial \tau} = a \left[\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \theta}{\partial r} \right]. \quad (1)$$

The boundary conditions are

$$\left. \begin{aligned} \lambda \frac{\partial \theta(r, \tau)}{\partial r} \Big|_{r=R} + \alpha \theta(r, \tau) \Big|_{r=R} &= 0 \\ \theta(r_0, \tau) &= \theta_c. \end{aligned} \right\} \quad (2)$$

The initial condition is

$$\theta(r, 0) = 0. \quad (3)$$

The solution is determined by separating the variables in the sum

$$\theta(r, \tau) = \theta_{tr}(r, \tau) + \theta_s(r).$$

The expression for θ_s is

$$\theta_s(\rho) = \theta_c \frac{1 - Bi \ln \rho}{1 - Bi \ln k}, \quad (4)$$

where

$$Bi = \frac{\alpha R}{\lambda}; \quad \rho = \frac{r}{R}; \quad k = \frac{r_0}{R}.$$

In turn,

$$\theta_{tr}(\rho, Fo) = \sum_{n=1}^{\infty} C_n H_n(\rho) \exp(-x_n^2 Fo), \quad (5)$$

where

$$H_n(\rho) = [\text{Bi } Y_0(x_n) - x_n Y_1(x_n)] J_0(x_n \rho) - [\text{Bi } J_0(x_n) - x_n J_1(x_n)] Y_0(x_n \rho),$$

and x_n are the roots of the characteristic equation

$$[\text{Bi } Y_0(x_n) - x_n Y_1(x_n)] J_0(x_n k) - [\text{Bi } J_0(x_n) - x_n J_1(x_n)] Y_0(x_n k) = 0.$$

The system of functions $H_n(\rho)$ is orthogonal with weight ρ [1].

The coefficients C_n are given by

$$C_n = \frac{\int_k^1 \rho H_n(\rho) \theta_s(\rho) d\rho}{\int_k^1 \rho H_n^2(\rho) d\rho}.$$

Table 1 gives the values for C_n .

We sum (4) and (5) to get a general formula for the temperature distribution:

$$\theta(\rho, \tau) = \sum_{n=1}^{\infty} C_n H_n(\rho) \exp(-x_n^2 \text{Fo}) + \theta_c \frac{1 - \text{Bi} \ln \rho}{1 - \text{Bi} \ln k}. \quad (6)$$

The hone 3 consists of a diamond-bearing layer containing the bonding agent 2 and the holder 1 (Fig. 1b), which are considered as two inhomogeneous plates in ideal thermal contact. At the surface $y=l$ a temperature θ_c is maintained, while at surface $y=0$ there is no heat transfer (we neglect the heat loss to the cone).

The solution of [1] has been used to determine the temperature of the hone.

In our case,

$$\frac{\theta_2(y, \text{Fo})}{\theta_c} = 1 - \sum_{n=1}^{\infty} \frac{2}{\mu_n \psi_n} \cos\left(\mu_n \frac{K_l}{\sqrt{K_a}} \cdot \frac{y}{l_1}\right) \exp\left(-\frac{\mu_n^2 K_l^2 \text{Fo}}{K_a}\right); \quad (7)$$

$$\frac{\theta_2(y, \text{Fo})}{\theta_c} = 1 - \sum_{n=1}^{\infty} \frac{2}{\mu_n \psi_n} \left[\cos\left(\mu_n \frac{y-l_1}{l-l_1}\right) \cos\left(\mu_n \frac{K_l}{\sqrt{K_a}}\right) - K_e \sin\left(\mu_n \frac{y-l_1}{l-l_1}\right) \sin\left(\mu_n \frac{K_l}{\sqrt{K_a}}\right) \right] \exp\left(-\frac{\mu_n K_l^2 \text{Fo}}{K_a}\right), \quad (8)$$

where

$$\psi_n = \left(1 + \frac{K_e K_l}{\sqrt{K_a}}\right) \sin \mu_n \cos \frac{\mu_n K_l}{\sqrt{K_a}} + \left(K_e + \frac{K_l}{\sqrt{K_a}}\right) \cos \mu_n \sin \frac{\mu_n K_l}{\sqrt{K_a}};$$

and μ_n are the roots of the characteristic equation;

$$K_e \text{tg } \mu_n \text{tg } \frac{\mu_n K_l}{\sqrt{K_a}} = 1;$$

$$K_a = \frac{a_2}{a_3}; \quad K_e = \frac{\lambda_2}{\lambda_3} \sqrt{\frac{a_3}{a_2}}; \quad K_l = \frac{l_1}{l-l_1}; \quad \text{Fo} = \frac{a\tau}{l_1^2}.$$

Figure 2 shows computer calculations, which correspond to machining a component made of SCh28-48 cast iron ($\alpha = 0.12 \text{ cal/cm} \cdot \text{sec} \cdot ^\circ\text{C}$) with the surface speed of the hone of $V_0 = 66 \text{ m/min}$, the speed of the reciprocating motion $V_1 = 9 \text{ m/min}$, specific pressure $p = 6 \text{ kgf/cm}^2$, and $\theta_c = 66^\circ\text{C}$, the latter being derived from experiment by means of foil thermocouples [2].

The ordinate in Fig. 2 is the dimensionless temperature θ/θ_c , while the abscissa is the dimensionless thickness of workpiece or hone. It is clear that the temperature distribution in the tool can be taken as of steady-state type under the conditions of the example for $\tau = 18 \text{ sec}$ ($\text{Fo} = 1.5$).

Figure 2 also shows that the value of Fo at which the temperature distribution may be considered as stationary falls comparatively rapidly as the relative thickness of the workpiece wall falls.

The deformation may be determined by considering the component as having an axially symmetrical temperature distribution with uniformly distributed radial pressure on the internal surface.

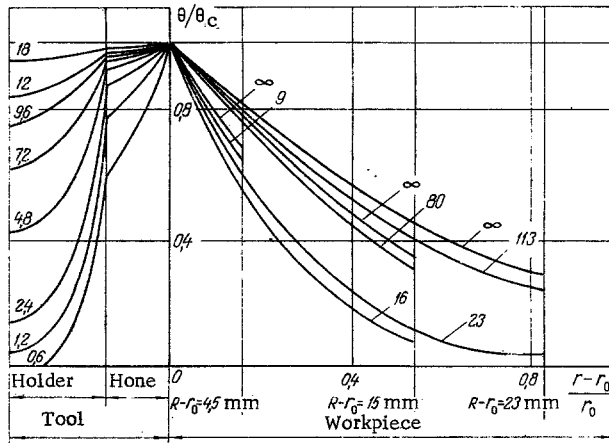


Fig. 2. Temperature distributions in tool and workpiece ($\alpha = 0.12 \text{ cal/cm}^2 \cdot \text{sec} \cdot ^\circ\text{C}$, $r_0 = 2.75 \text{ cm}$). The numbers on the curves are the values of τ .

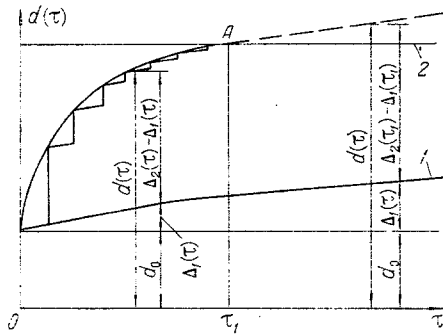


Fig. 3. Schematic representation of metal removal in the presence of thermal expansion in the tool.

If we use the formulas of [3], we get an expression for the deformation of the internal diameter in the form

$$\Delta_1(\tau) = \Delta d_0 = 2 \left(-\frac{\varepsilon_{zz}}{2} r_0 + \frac{c}{r_0} \right), \quad (9)$$

where

$$c = \frac{p + 2E \int_0^R \frac{1}{\eta} [\alpha_1^* \theta(\eta, \tau) - 2J(\eta)] d\eta}{\frac{2}{3} E \left(\frac{1}{r_0^2} - \frac{1}{R^2} \right)}$$

Here

$$J(\eta) = \frac{1}{\eta^2} \int_0^\eta \xi \alpha_1^* \theta(\xi, \tau) d\xi,$$

and ε_{zz} is a component of the strain tensor:

$$\varepsilon_{zz} = \frac{2R^2 J(R)}{R^2 - r^2}.$$

The deformation in the hone may be calculated from the formula for the strain in a one-dimensional rod:

$$\Delta_2(\tau) = \int_0^l \alpha^*(y) \theta(y, \tau) dy.$$

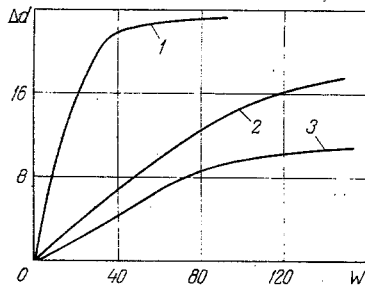


Fig. 4. Thermal determination Δd (μm) of workpiece (cast iron) in relation to metal removed W (μm) for workpieces of wall thickness (mm): 1) 4.5; 2) 15; 3) 23.

The deformation in the hone in response to the radial force can be neglected, since the force in honing is comparatively small, and, consequently, the strain is extremely small.

We assume that α_2^* and α_3^* are the coefficients of linear expansion for the holder and hone, which gives us a formula for the temperature deformation of the hone in the form

$$\Delta_2(\tau) = \int_0^{l_1} \alpha_2^* \theta_2(y, \tau) dy + \int_{l_1}^l \alpha_3^* \theta_3(y, \tau) dy. \quad (10)$$

We are interested in the total thermal strain arising from interaction of the strains in the workpiece and tool with a view to installation of an automatic monitoring device; these quantities can be derived by computer calculation from (9) and (10).

The results show that the rate of thermal expansion in the hone is initially higher than that in the workpiece in most cases. Thin-walled components represent an exception.

We may represent honing schematically as removal of metal in discrete form in each double pass of the tool; Fig. 3 shows the thermal expansion of the workpiece (curve 1) and that of the hone (curve 2). The steps show the discrete changes in the internal diameter of the workpiece, with allowance for the metal removed on account of the difference in the thermal expansion.

It is clear that there are two clearly defined periods, in the first of which the rate of expansion of the tool exceeds that of the workpiece and the cutting rate increases continuously. At some instant τ_1 the rates of expansion of tool and workpiece come to balance, whereupon the second period starts, where the expansion of the workpiece exceeds that of the tool.

Under these conditions, the cutting continues only on account of the radial feed of the tool, which is provided by the mechanism.

Then the change in internal diameter due to the thermal expansion of the tool and workpiece can be put as

$$d(\tau) = \begin{cases} d_0 + \Delta_2(\tau) & \tau < \tau_1 \\ d_0 + \Delta_2(\tau_1) - \Delta_1(\tau_1) + \Delta_1(\tau), & \tau > \tau_1, \end{cases}$$

where d_0 is the initial internal diameter. The amount of metal removed on account of the thermal expansion is

$$w_0(\tau) = d(\tau) - d_0 - \Delta_1(\tau), \\ w_0(\tau) = \begin{cases} \Delta_2(\tau) - \Delta_1(\tau), & \tau < \tau_1, \\ \Delta_2(\tau_1) - \Delta_1(\tau_1), & \tau > \tau_1. \end{cases}$$

The specific cutting rate is

$$Q(\tau) = \begin{cases} \frac{d\Delta_2(\tau)}{d\tau} - \frac{d\Delta_1(\tau)}{d\tau}, & \tau < \tau_1, \\ 0, & \tau > \tau_1. \end{cases}$$

The latter rises initially as a function of time but then falls; various studies [4-6] explain this behavior of the cutting curve, as well as the accelerated removal of the initial roughness and shape errors in the workpiece. On the other hand, careful experiments that eliminate the above errors do show that this turning-point behavior is still present, so the effect is due not only to the initial errors, but also to the different rates of expansion of the tool and workpiece.

In the first period of honing, the rate of increase in the thermal deformation of the hone is substantially greater than that for the workpiece, which accelerates the cutting. Subsequently, the rates of expansion become equal, and then ultimately the workpiece expands more rapidly than the tool, so the cutting rate falls.

The size of the workpiece obtained as a result of honing subsequently alters during cooling.

Since honing is a comparatively rapid process, it may be completed before the steady thermal state is reached, in which case it is necessary to determine the workpiece deformation in relation to the amount of metal removed.

We can determine the latter from formulas given in [4]:

$$w(\tau) = \begin{cases} R_{zini} \left[1 - \exp\left(-\frac{u\tau}{R_{zfin}}\right) \right], & \tau < \tau_{fin}, \\ R_{zini} \left[1 - \exp\left(-\frac{u\tau_{fin}}{R_{zfin}}\right) \right] + u(\tau - \tau_{fin}), & \tau > \tau_{fin}. \end{cases} \quad (11)$$

We substitute $\tau_{fin} = (R_{zfin}/u) \ln(R_{zini}/R_{zfin})$ in (11) and make various simplifications to get

$$w(\tau) = \begin{cases} R_{zini} \left[1 - \exp\left(-\frac{u\tau}{R_{zfin}}\right) \right], & \tau < \tau_{fin}, \\ (R_{zini} - R_{zfin}) + u(\tau - \tau_{fin}), & \tau > \tau_{fin}. \end{cases}$$

Then the expression for the total amount of metal removed, as corrected for the thermal expansion of the hone, is

$$W(\tau) = w(\tau) + w_0(\tau).$$

These calculations on the thermal expansion of the workpiece and the metal cut have provided the former as a function of the latter for various workpiece wall thicknesses (Fig. 4).

These and similar graphs derived as above allow one to correct for the thermal expansion of the workpiece, and thus substantially improve the accuracy of monitoring and thus of the workpieces themselves.

NOTATION

a_1, a_2, a_3 , thermal diffusivities of workpiece, holder, and hone, respectively; $\lambda_1, \lambda_2, \lambda_3$, thermal conductivities of workpiece, holder, and hone, respectively; α , heat-transfer coefficient for the outer surface of workpiece; $\theta_1, \theta_2, \theta_3$, temperatures of workpiece, holder, and hone, respectively; θ_c , contact temperature; r_0, R , inner and outer radii of workpiece; $\alpha_1^*, \alpha_2^*, \alpha_3^*$, coefficients of linear expansion of workpiece, holder, and hone, respectively; E , elastic modulus of workpiece; ρ , dimensionless radius; Bi, Fo , dimensionless Biot and Fourier numbers; Δ_1, Δ_2 , thermal increase in the internal diameter of workpiece and in tool diameter; Q , specific amount of metal cut; R_{zini}, R_{zfin} , initial and final heights of microroughness of workpiece surface; τ , treatment time; τ_{fin} , end of removal of the initial roughness; w , metal removed by radial feed; w_θ , metal removed on account of thermal expansion of tool; Δd_0 , error in the internal diameter of workpiece due to expansion; p , specific contact pressure between hone and workpiece; θ_{tr}, θ_s , transient and steady-state components of temperature distribution, °C; u , base metal removed.

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